



Semester One Examination, 2020

Question/Answer booklet

**MATHEMATICS
METHODS
UNIT 3**

**Section One:
Calculator-free**

If required by your examination administrator, please
place your student identification label in this box

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	99	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (50 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

A curve passes through the point $A(2, 1)$ and its gradient is given by

$$\frac{dy}{dx} = 3x^2 - \frac{8}{x^2} - 10$$

- (a) Verify that A is a stationary point, determine the value of the second derivative at A and hence describe the nature of the stationary point. (3 marks)

- (b) Determine the equation of the curve. (2 marks)

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Question 2

(5 marks)

Determine the area bounded by the line $y = -2x$ and the parabola $y = x^2 - 6x$.

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Question 3

(8 marks)

Determine

(a) $f'(x)$ when $f(x) = \sqrt{4x - 3}$. (2 marks)

(b) $\frac{d}{dm}(m^3 e^{4m})$ when $m = 2$. (3 marks)

(c) $f'\left(\frac{\pi}{4}\right)$ when $f(t) = \frac{1 + \cos t}{\sin t}$. (3 marks)

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Question 4

(7 marks)

A bag contains 40 counters, 15 marked with 0 and the remainder marked with 1. The random variable X is the number on a randomly selected counter from the bag.

- (a) Explain why X is a Bernoulli random variable and determine the mean and variance of X .
(3 marks)

Each of the 32 students in a class randomly select a counter from the bag, note the number on the counter and then replace it back in the bag. The random variable Y is the number of students in the class who select a counter marked with 0.

- (b) Define the distribution of Y and determine the mean and variance of Y .
(3 marks)

- (c) Explain why it is important that the students replace their counters for the distribution of Y in part (b) to be valid.
(1 mark)

Question 5

(5 marks)

A discrete uniform distribution, X , has outcomes 0 to n and has an expected value, $E(X) = 5.5$.

(a) Calculate n , the number of outcomes for X . (2 marks)

(b) Write the probability distribution function for X . (2 marks)

(c) Suggest a possible experiment for which the distribution of X could represent. (1 mark)

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Question 6

(5 marks)

The graph of $y = f(x)$ has a stationary point at $(2, 5)$ and $f'(x) = ax^2 - 9x + 6$, where a is a constant.

Determine the interval over which $f'(x) < 0$ and $f''(x) < 0$.

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Question 7

(8 marks)

Initially, particle P is stationary and at the origin. Particle P moves in a straight line so that at time t seconds its acceleration a cms^{-2} is given by $a = 8 - 3\sqrt{t}$ where $t \geq 0$.

(a) Determine the speed of P after 1 second. (3 marks)

(b) Determine the speed of P when it returns to the origin. (5 marks)

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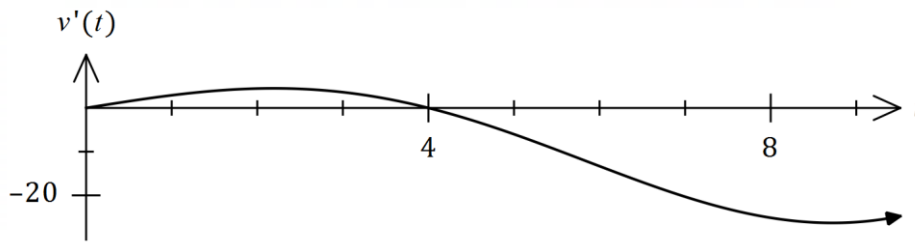
Question 8

(7 marks)

(a) Determine an expression for $\frac{d}{dt}\left(8t \sin\left(\frac{\pi t}{8}\right)\right)$.

(2 marks)

The volume of water in a tank, v litres, is changing at a rate given by $v'(t) = \pi t \cos\left(\frac{\pi t}{8}\right)$, where t is the time in hours. The rate of change is shown in the graph below.



(b) Using the result from part (a) or otherwise, determine the change in volume of water in the tank between $t = 0$ and $t = 8$ hours. (5 marks)

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Supplementary page

Question number: _____

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